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AESB2341
Rock Mechanics and Rock Engineering

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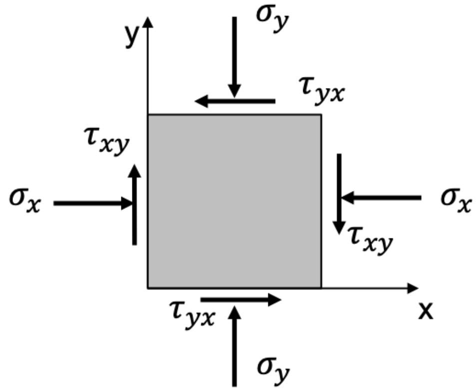
Formulae sheet

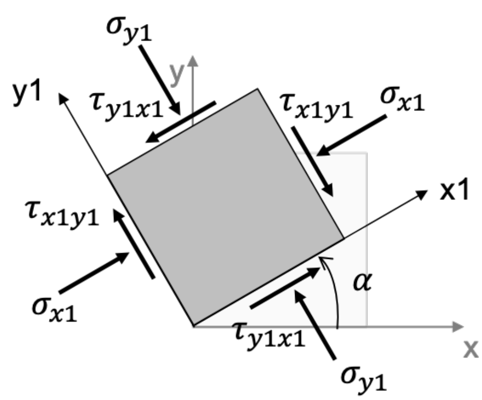
Hooke's law

$$\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ 2\varepsilon_{12}\\ 2\varepsilon_{23}\\ 2\varepsilon_{13}\end{bmatrix} = \frac{1}{E}\begin{bmatrix}1 & -\nu & -\nu & 0 & 0 & 0\\ -\nu & 1 & -\nu & 0 & 0 & 0\\ -\nu & -\nu & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0\\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0\\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu)\end{bmatrix}\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\\ \sigma_{23}\\ \sigma_{13}\end{bmatrix} \tag{1}$$

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\\ \sigma_{23}\\ \sigma_{13}\end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & \nu & 0 & 0 & 0\\ \nu & 1-\nu & \nu & 0 & 0 & 0\\ \nu & \nu & 1-\nu & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ 2\varepsilon_{12}\\ 2\varepsilon_{23}\\ 2\varepsilon_{13}\end{bmatrix} \tag{2}$$

Transformation equations





$$\sigma_{x1} = \sigma_x \cos^2\alpha + \sigma_y \sin^2\alpha + 2\tau_{xy}\sin\alpha\cos\alpha \tag{3a}$$

$$\sigma_{y1} = \sigma_x \sin^2\alpha + \sigma_y \cos^2\alpha - 2\tau_{xy}\sin\alpha\cos\alpha \tag{3b}$$

$$\tau_{x1y1} = \tau_{xy}\left(\cos^2\alpha - \sin^2\alpha\right) - (\sigma_x - \sigma_y)\sin\alpha\cos\alpha \tag{3c}$$

Principal stresses

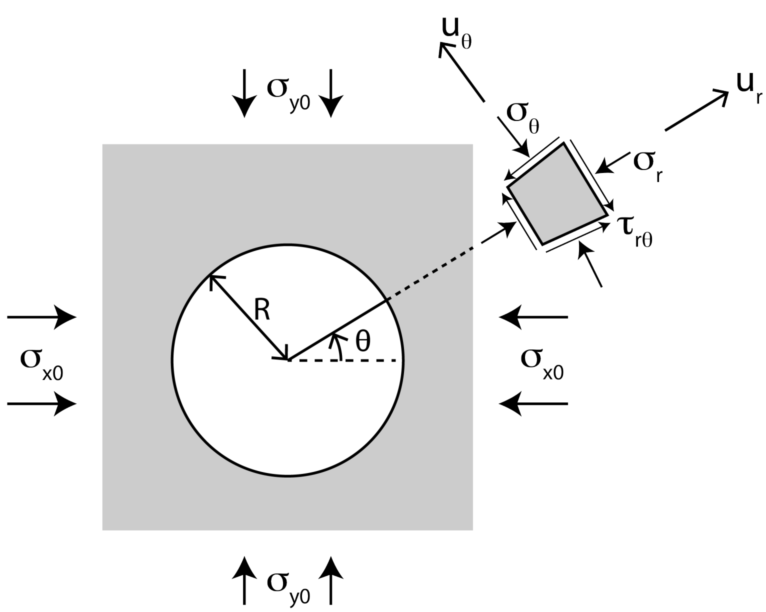
$$\sigma_1 = \frac{1}{2}(\sigma_x + \sigma_y) + \sqrt{\tau_{xy}^2 + \frac{1}{4}(\sigma_x - \sigma_y)^2} \tag{4a}$$

$$\sigma_3 = \frac{1}{2}(\sigma_x + \sigma_y) - \sqrt{\tau_{xy}^2 + \frac{1}{4}(\sigma_x - \sigma_y)^2} \tag{4b}$$

$$\alpha = \begin{cases} \frac{\delta}{2} & \text{if } \sigma_x > \sigma_y \\ \frac{\delta}{2} + \frac{\pi}{2} & \text{if } \sigma_x < \sigma_y \text{ and } \tau_{xy} > 0 \\ \frac{\delta}{2} - \frac{\pi}{2} & \text{if } \sigma_x < \sigma_y \text{ and } \tau_{xy} < 0 \end{cases} \tag{5}$$

$$\delta = \arctan\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \tag{6}$$

Kirsch equations



Stress state :

$$\sigma_r = \frac{\sigma_{x0} + \sigma_{y0}}{2}\left(1 - \frac{R^2}{r^2}\right) + \frac{\sigma_{x0} - \sigma_{y0}}{2}\left(1 - \frac{4R^2}{r^2} + \frac{3R^4}{r^4}\right)\cos 2\theta \tag{7a}$$

$$\sigma_\theta = \frac{\sigma_{x0} + \sigma_{y0}}{2}\left(1 + \frac{R^2}{r^2}\right) - \frac{\sigma_{x0} - \sigma_{y0}}{2}\left(1 + \frac{3R^4}{r^4}\right)\cos 2\theta \tag{7b}$$

$$\tau_{r\theta} = -\frac{\sigma_{x0} - \sigma_{y0}}{2}\left(1 + \frac{2R^2}{r^2} - \frac{3R^4}{r^4}\right)\sin 2\theta \tag{7c}$$

Displacements :

$$u_r = \frac{\sigma_{x0} + \sigma_{y0}}{4G}\frac{R^2}{r^2} + \frac{\sigma_{x0} - \sigma_{y0}}{4G}\frac{R^2}{r^2}\left[4\left(1-\nu\right) - \frac{R^2}{r^2}\right]\cos 2\theta \tag{8a}$$

$$u_\theta = -\frac{\sigma_{x0} - \sigma_{y0}}{4G}\frac{R^2}{r^2}\left[2\left(1-2\nu\right) + \frac{R^2}{r^2}\right]\sin 2\theta \tag{8b}$$

$$G = \frac{E}{2\left(1+\nu\right)} \tag{9}$$

Bearing capacity

$$q_b = \frac{1}{2}\gamma w_p N_\gamma S_\gamma + c\cot\phi N_q S_q - c\cot\phi \tag{10}$$

with :

	Shape factors ($l_p > w_p$) :
$N_\gamma = 1.5\left(N_q - 1\right)\tan\phi$	$N_q = \exp\left(\pi\tan\phi\right)\tan^2\left(\pi/4 + \phi/2\right)$
$S_\gamma = 1 - 0.4\left(w_p/l_p\right)$	$S_q = 1 + \sin\phi\left(w_p/l_p\right)$

Goodman model

$$\frac{\sigma_n - \sigma_{ni}}{\sigma_{ni}} = A\left(\frac{\Delta u_n}{\Delta u_{n,max} - \Delta u_n}\right)^t \tag{11}$$

JRC-JCS empirical model

$$\tau = \sigma_n' \tan\left[JRC\log_{10}\left(\frac{JCS}{\sigma_n}\right) + \phi_r\right] \tag{12}$$

Hydraulic aperture

$$d_e = JRC^{2,5}\left(\frac{d}{d_e}\right)^{-2} \tag{13}$$

Trigonometric identities

$$\sin\left(2\beta\right) = 2\sin\beta\cos\beta \tag{14}$$

$$\cos\left(2\beta\right) = \cos^2\beta - \sin^2\beta \tag{15}$$